On Modeling Multi-Agent Task Scheduling as a Distributed Constraint Optimization Problem

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Scenario: Hostage Rescue

Goal: Create a Schedule
- Mitigate Crisis
- Capture Terrorists
- Save Hostages

Coast Guard
Philadelphia Police
Surveil
Arrest
Camden Police
Extract
Camden EMT
Treat

Synchronization Point
Hostages Killed

On Modeling Multi-Agent Task Scheduling
Scenario: Hostage Rescue

- Goal: Create a Schedule, Mitigate Crisis, Capture Terrorists, Save Hostages
- Participants: Coast Guard, Philadelphia Police, Camden Police, Camden EMT

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On Modeling Multi-Agent Task Scheduling
Scenario: Hostage Rescue

The Problem of Coordination

- Goal: Create a Schedule
- Mitigate Crisis
- Capture Terrorists
- Save Hostages

Involving:
- Coast Guard
- Philadelphia Police
- Surveil
- Arrest
- Camden Police
- Extract
- Camden EMT
- Treat

Synchronization Point:
- Hostages
- Killed

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On Modeling Multi-Agent Task Scheduling
Scenario: Hostage Rescue

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Synchronization Point

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Scenario: Hostage Rescue

Goal: Create a Schedule, Mitigate Crisis, Capture Terrorists, Save Hostages

Coast Guard, Philadelphia Police, Surveil, Arrest
Camden Police, Extract
Camden EMT, Treat

Synchronization Point
Hostages, Killed

On Modeling Multi-Agent Task Scheduling
Scenario: Hostage Rescue

Goal: Create a Schedule
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Synchronization Point
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On Modeling Multi-Agent Task Scheduling
Scenario: Hostage Rescue

Goal: Create a Schedule

Mitigate Crisis
Capture Terrorists
- Coast Guard: Surveil
- Philadelphia Police: Arrest

Save Hostages
- Camden Police
- Camden EMT: Extract, Treat

Synchronization Point
Hostages Killed

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Goal: Create a Schedule

Mitigate Crisis

Capture Terrorists

Save Hostages

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Camden Police

Extract

Camden EMT

Treat

Synchronization

Point

Hostages

Killed

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On Modeling Multi-Agent Task Scheduling
Scenario: Hostage Rescue

Domain Challenges:
- Distribution
- Interaction between different organizations & jurisdictions.
- Heterogeneous communication systems.
- Temporal dependencies; actions of one agent can enable/disable facilitate/hinder actions of another.

Technical Challenges:
- Representation
- Algorithms
- Optimizations (i.e., making the algorithms feasible)
- Measurement
Domain Challenges

- **Distribution**
  - Interaction between different organizations & jurisdictions.
  - Heterogeneous communication systems.

- **Temporal dependencies**; actions of one agent can...
  - enable/disable
  - facilitate/hinder

...actions of another
Domain Challenges

- Distribution
  - Interaction between different organizations & jurisdictions.
  - Heterogeneous communication systems.
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Technical Challenges

- Representation
- Algorithms
- Optimizations (i.e., making the algorithms feasible)
- Measurement

Scenario: Hostage Rescue

Domain Challenges

Scenario

The Problem of Coordination

On Modeling Multi-Agent Task Scheduling
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Task Scheduling Problem

Representation

\[ C_{TÆMS} \]

Algorithms

DCOP

SCHEDULE

Optimizations

Measurement

Macro Agent Transport Event-Based Simulator (MATES)

Department of Computer Science

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On Modeling Multi-Agent Task Scheduling
**Approach**

**Task Scheduling Problem**

**Representation**

$\text{C}_\text{TÆMS}$

**Algorithms**

DCOP

**Schedule**

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Algorithms
Optimizations

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**Approach**

**Task Scheduling Problem**

**Representation**

\[ \mathcal{C}_{\text{TÆMS}} \]

**Algorithms**

\[ \text{DCOP} \]

**Optimizations**

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**Task Scheduling Problem**

**Representation**

C_TÆMS

**Algorithms**

DCOP

**Schedule**

Optimizations Measurement

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On Modeling Multi-Agent Task Scheduling
Discrete, stochastic modeling language (formal grammar)\(^1\).

Derivative of the TÆMS modeling language\(^2\).

- Defined in the context of the COORDINATORS Project.

Hierarchical Task Network\(^3\).

Widely used in the multi-agent systems community\(^4\)\(^5\)\(^6\)\(^7\).

\(^1\) Boddy, et al., 2006
\(^2\) K. Decker, 1996
\(^3\) K. Erol, J. Hendler, and D. Nau, 1994
\(^4\) Musliner, Durfee, Wu, Dolgov, Goldman, and Boddy, 2006
\(^5\) Smith, Gallagher, Zimmerman, Barbulescu, and Rubinstein, 2006
\(^6\) Phelps and Rye, 2006
\(^7\) Cornell C_TÆMS Scheduler
Methods/Tasks: like actions
Non-Local Effects (NLEs): temporal constraints
NLE Chains
⟨Earliest Start Time, Latest Start Time, Expected Duration⟩
Quality Accumulation Functions (QAFs)
Goal: create a schedule that maximizes quality at the root.
Approach

**Task Scheduling Problem**

**Representation**

- $C_{TÆMS}$

**Algorithms**

- DCOP

**Schedule**

**Optimizations**

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**TASK SCHEDULING PROBLEM**

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Definition

* Distributed Constraint Optimization Problem *

- Each agent has a set of variables that need assignment;
- Each variable has a finite *domain* containing possible values;
- Set of constraints between agents’ variables’.

- Constraints are a natural way to represent dependencies between agent actions.
- There exist guaranteed-optimal *distributed* algorithms for DCOP:
  - DPOP (Petcu and Faltings, 2005);
  - OptAPO (Mailler and Lesser, 2004); and
**DCOP**

**Definition**

*Distribution Optimization Problem*:
- Each agent has a set of variables that need assignment;
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**Definition**
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  - ADOPT (Modi, et al., 2005).
**Definition**

*Distribution Constraint Optimization Problem (DCOP)*

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  - ADOPT (Modi, et al., 2005).

**Example**

```
A {R,G,B}  ≠ ≠ ≥ ≤
C {R,G,B}  ≠ ≠ ≥ ≤
B {R,G,B}  ≠ ≠ ≥ ≤
D {R,G,B}  ≠ ≠ ≥ ≤
```
DCOP

Definition

Distributed Constraint Optimization Problem (DCOP)

Example

Agent 1
Agent 2
Agent 3

Constraints between agents' variables:
Set of constraints in the problem

Representation

Algorithms

Optimization

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\( C_{\text{TÆMS}} \)

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Macro Agent Transport Event-Based Simulator (MATES)
Mapping \( \text{DSC\_TÀEMS} \rightarrow \text{DCOP} \)

**DCOP-Solvable C\_TÀEMS:** \( \text{DSC\_TÀEMS} \)

Subset of \( \text{C\_TÀEMS} \) that is solvable using existing DCOP algorithms.

- Expected value instead of prob. distributions;
- Only \( \sum \) QAFs;
- Only Enables/Disables NLEs; and
- Discretize time.
Mapping DSC\_\text{TAEMS} \rightarrow DCOP

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Mapping DSC\_\text{TAEMS} \rightarrow DCOP

- Mitigate Crisis
- Save Hostages
- Capture Terrorists
- Extract Hostages
- Medical Attention
- Arrest
- Surveil

Quality: 100

Quality: 100

Quality: 50

Quality: 10

<15, \text{\textlangle}15,60,30\rangle>

<25, \text{\textlangle}10,60,60\rangle>

<0,60,?>

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Mapping DSC\_TÆMS → DCOP

Methods ↔ Variables

Quality: 100  Quality: 100  Quality: 50  Quality: 10

Mitigate Crisis

Save Hostages

Capture Terrorists

<15,60,30>

<0,60,?>

<10,60,60>

<25,10>

<15,15>

Mitigate Crisis

Save Hostages

Capture Terrorists

Extract Hostages

Medical Attention

Arrest

Surveil

<15,15>

<25,10>

<15,60,30>

<10,60,60>

Quality: 100  Quality: 50  Quality: 10

Mitigate Crisis

Save Hostages

Capture Terrorists

Extract Hostages

Medical Attention

Arrest

Surveil

<15,15>

<25,10>

<15,60,30>

<10,60,60>

Quality: 100  Quality: 50  Quality: 10

Mitigate Crisis

Save Hostages

Capture Terrorists

Extract Hostages

Medical Attention

Arrest

Surveil

<15,15>

<25,10>

<15,60,30>

<10,60,60>

Quality: 100  Quality: 50  Quality: 10

Mitigate Crisis

Save Hostages

Capture Terrorists

Extract Hostages

Medical Attention

Arrest

Surveil

<15,15>

<25,10>

<15,60,30>

<10,60,60>
Mapping DSC_{TÆMS} → DCOP

Earliest/Latest Start Times ↦ Domains

Mitigate Crisis

Save Hostages

Capture Terrorists

Extract Hostages
Quality: 100

Medical Attention
Quality: 100

Arrest
Quality: 50

Surveil
Quality: 10

Extract Hostages
(Ø,15,16,...,∞)

Arrest
(Ø,15,16,...,60)

Medical Attention
(Ø,25,26,...,∞)

Surveil
(Ø,10,11,...,60)

Mitigate

Save

Capture

Hostages

Terrorists

<0,60,7>

<15,6,15>

<15,60,30>

<10,60,60>

<25,6,10>

<15,16,...,60>

(Ø,15,16,...,∞)

Discretize time.

Expected value instead of prob. distributions;

Only ∑QAFs;

Only Enables/Disables NLEs; and

DCOP-Solvable C_{TÆMS}: DSC_{TÆMS} Subset of C_{TÆMS} that is solvable using existing DCOP algorithms.
Mapping \( \text{DSC}_\text{TÆMS} \rightarrow \text{DCOP} \)

Qualities and QAFs \(\mapsto\) Soft Unary Constraints

- Mitigate Crisis
- Save Hostages
- Capture Terrorists
- Extract Hostages
- Medical Attention
- Arrest
- Surveil

Quality: 100
Quality: 100
Quality: 50
Quality: 10

- Philadelphia
- Camden
- Coast Guard

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**Mapping DSC\_TÆMS \rightarrow DCOP**

**Max Requires n-Ary Constraints**

- **Mitigate Crisis**
  - Save Hostages
  - Capture Terrorists

- **Save Hostages**
  - Extract Hostages
  - Medical Attention
  - Arrest
  - Surveil

- **Capture Terrorists**
  - Medical Attention
  - Surveil

- **Extract Hostages**
  - Quality: 100

- **Medical Attention**
  - Quality: 100

- **Arrest**
  - Quality: 50

- **Surveil**
  - Quality: 10

- **Philadelphia**
  - Camden
  - Coast Guard

- **Extract Hostages**
  - \(<15,15>\)

- **Medical Attention**
  - \(<25,10>\)

- **Arrest**
  - \(<15,60,30>\)

- **Surveil**
  - \(<10,60,60>\)

- **Extract Hostages**
  - \(<0,60,?>\)

- **Arrest**
  - \(<0,60,?>\)

- **Medical Attention**
  - \(<0,60,?>\)

- **Surveil**
  - \(<0,60,?>\)

- **DCOP-Solvable C\_TÆMS**: DSC\_TÆMS Subset of C\_TÆMS that is solvable using existing DCOP algorithms.

- **Expected value instead of prob. distributions**
- **Only \(\sum\)** QAFs
- **Only Enables/Disables NLEs**
- **Discretize time**

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Mapping DSC\_TÆMS → DCOP

Enables/Disables NLEs ↔ Hard Binary Constraints
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Mapping $\text{DSC}_\text{TÆMS} \rightarrow \text{DCOP}$

Facilitates/Hinders Require $n$-Ary Constraints

- Mitigate Crisis
- Save Hostages
- Capture Terrorists
- Extract Hostages
- Medical Attention
- Arrest
- Surveil

$\Sigma$

$\text{Extract Hostages}$
$\text{Medical Attention}$
$\text{Arrest}$
$\text{Surveil}$

$\text{Mitigate Crisis}$

$\text{Save Hostages}$

$\text{Capture Terrorists}$

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Mapping $\text{DSC}_{\text{TÆMS}} \rightarrow \text{DCOP}$

Agents $\rightarrow$ Agents
Mapping DSC \_TAEMS $\rightarrow$ DCOP

Mutex Constraints

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On Modeling Multi-Agent Task Scheduling
Theorem

The problem of finding an optimal schedule for a DSC_TÆMS instance is $\mathcal{NP}$-HARD.

Corollary

Even if all of the variables’ domains are of cardinality two, the problem is still $\mathcal{NP}$-HARD!
The problem of finding an optimal schedule for a DSC\textsubscript{TaEMS} instance is \(\mathcal{NP}\)\text{-HARD}.

Even if all of the variables’ domains are of cardinality two, the problem is still \(\mathcal{NP}\)\text{-HARD}!
**Approach**

**Task Scheduling Problem**

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**Task Scheduling Problem**

- **Representation**
  - C_TÆMS

- **Algorithms**
  - DCOP

- **Schedule**

**Optimizations**

- Measurement
  - Macro Agent Transport Event-Based Simulator (MATES)

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Domain Size Reduction

- Finding optimal DCOP solutions has computational complexity exponential in the average domain size.
- Ideally, we want to minimize domain sizes, (or at least make them finite).
Finding optimal DCOP solutions has computational complexity exponential in the average domain size.

Ideally, we want to minimize domain sizes, (or at least make them finite).

**Problem:** Need ways to reduce domain size!
Naïve Domain Bounding

We can define an upper bound on the start time of a method as the maximum finite earliest start time among all methods plus the duration of all other methods.

**Intuition**

In the extreme, the method with maximum finite earliest start time will be executed *first*, with all other methods proceeding in succession.

**Theorem**

*This method of naïve domain bounding will not result in suboptimal solutions.*
Naïve Domain Bounding

We can define an upper bound on the start time of a method as the maximum finite earliest start time among all methods plus the duration of all other methods.

Intuition

In the extreme, the method with maximum finite earliest start time will be executed first, with all other methods proceeding in succession.

Theorem

*This method of naïve domain bounding will not result in suboptimal solutions.*
Example

\[ M = \{ M_1 = \text{Extract} = \langle 15, \infty, 15 \rangle, \]
\[ M_2 = \text{Medical} = \langle 25, \infty, 10 \rangle, \]
\[ M_3 = \text{Arrest} = \langle 15, 60, 30 \rangle, \]
\[ M_4 = \text{Surveil} = \langle 10, 60, 60 \rangle \} \]

\[ u'_{M_1} = \max \{ u_{M_3} = 60, u_{M_4} = 60 \} \]
\[ + \sum \{ d_{M_1} = 15, d_{M_2} = 10, d_{M_3} = 30, d_{M_4} = 60 \} \]
\[ = 175. \]
Example

\[ \mathcal{M} = \{ M_1 = \text{Extract} = \langle 15, \infty, 15 \rangle, \]
\[ M_2 = \text{Medical} = \langle 25, \infty, 10 \rangle, \]
\[ M_3 = \text{Arrest} = \langle 15, 60, 30 \rangle, \]
\[ M_4 = \text{Surveil} = \langle 10, 60, 60 \rangle \} \]

\[ \Downarrow \]

\[ u'_{M_1} = \max \{ u_{M_3} = 60, u_{M_4} = 60 \} \]
\[ + \sum \{ d_{M_1} = 15, d_{M_2} = 10, d_{M_3} = 30, d_{M_4} = 60 \} \]
\[ = 175. \]
Example

\[ M = \{ M_1 = \text{Extract} = \langle 15, 175, 15 \rangle, \]
\[ M_2 = \text{Medical} = \langle 25, \infty, 10 \rangle, \]
\[ M_3 = \text{Arrest} = \langle 15, 60, 30 \rangle, \]
\[ M_4 = \text{Surveil} = \langle 10, 60, 60 \rangle \} \]

\[ u'_{M_1} = \max \{ u_{M_3} = 60, u_{M_4} = 60 \} \]
\[ + \sum \{ d_{M_1} = 15, d_{M_2} = 10, d_{M_3} = 30, d_{M_4} = 60 \} \]
\[ = 175. \]
It is possible to improve upon the naïve mapping by exploiting the hierarchy of DSC\textsubscript{TÆMS}' structure.

**Intuition**

Children’s bounds must be at least as restrictive as their parents’.
Syntax: 〈Earliest Start Time, Latest Start Time〉

Example

Mitigate Crisis 〈0, 75〉

Save Hostages 〈10, 30〉

Capture Terrorists 〈20, 100〉

Extract Hostages 〈0, ∞〉

Medical Attention 〈0, ∞〉

Arrest 〈0, 50〉

Surveil 〈0, 45〉

Average Domain Size in DCOP: ∞
Syntax: $\langle$Earliest Start Time, Latest Start Time$\rangle$

Example

Mitigate Crisis
$\langle 0, 75 \rangle$

Save Hostages
$\langle 10, 30 \rangle$

Capture Terrorists
$\langle 20, 100 \rangle$

Extract Hostages
$\langle 0, \infty \rangle$

Medical Attention
$\langle 0, \infty \rangle$

Arrest
$\langle 0, 50 \rangle$

Surveil
$\langle 0, 45 \rangle$

Average Domain Size in DCOP: $\infty$
Syntax: \( \langle \text{Earliest Start Time, Latest Start Time} \rangle \)

Example

- **Mitigate Crisis** \( \langle 0, 75 \rangle \)
  - **Save Hostages** \( \langle 10, 30 \rangle \)
  - **Capture Terrorists** \( \langle 20, 100 \rangle \)
    - **Extract Hostages** \( \langle 10, \infty \rangle \)
    - **Medical Attention** \( \langle 0, \infty \rangle \)
    - **Arrest** \( \langle 0, 50 \rangle \)
    - **Surveil** \( \langle 0, 45 \rangle \)

Average Domain Size in DCOP: \( \infty \)
Syntax: \(\langle \text{Earliest Start Time, Latest Start Time} \rangle\)

Example

```
Mitigate Crisis
\(\langle 0, 75 \rangle\)
```

```
Save Hostages
\(\langle 10, 30 \rangle\)
```

```
Capture Terrorists
\(\langle 20, 100 \rangle\)
```

```
Save Hostages
\(\langle 10, ∞ \rangle\)
```

```
Save Hostages
\(\langle 10, ∞ \rangle\)
```

```
Medical Attention
\(\langle 0, ∞ \rangle\)
```

```
Arrest
\(\langle 0, 50 \rangle\)
```

```
Surveil
\(\langle 0, 45 \rangle\)
```

Average Domain Size in DCOP: \(∞\)
Syntax: \(\langle\text{Earliest Start Time, Latest Start Time}\rangle\)

**Example**

Mitigate Crisis
\(\langle 0, 75 \rangle\)

- Save Hostages
  - Extract Hostages
    - \(\langle 10, 30 \rangle\)
  - Medical Attention
    - \(\langle 0, \infty \rangle\)

- Capture Terrorists
  - Arrest
    - \(\langle 0, 50 \rangle\)
  - Surveil
    - \(\langle 0, 45 \rangle\)

Average Domain Size in DCOP: \(\infty\)
Syntax: \( \langle \text{Earliest Start Time, Latest Start Time} \rangle \)

Example

Mitigate Crisis
\( \langle 0, 75 \rangle \)

Save Hostages
\( \langle 10, 30 \rangle \)

Capture Terrorists
\( \langle 20, 100 \rangle \)

Extract Hostages
\( \langle 10, 30 \rangle \)

Medical Attention
\( \langle 0, \infty \rangle \)

Arrest
\( \langle 0, 50 \rangle \)

Surveil
\( \langle 0, 45 \rangle \)

Average Domain Size in DCOP: \( \infty \)
Syntax: \(\langle \text{Earliest Start Time, Latest Start Time} \rangle\)

Example

- **Mitigate Crisis**: \(\langle 0, 75 \rangle\)
  - **Save Hostages**: \(\langle 10, 30 \rangle\)
    - **Extract Hostages**: \(\langle 10, 30 \rangle\)
    - **Medical Attention**: \(\langle 10, \infty \rangle\)
  - **Capture Terrorists**: \(\langle 20, 100 \rangle\)
    - **Arrest**: \(\langle 0, 50 \rangle\)
    - **Surveil**: \(\langle 0, 45 \rangle\)

**Average Domain Size in DCOP**: \(\infty\)
Syntax: $\langle$Earliest Start Time, Latest Start Time$\rangle$

Example

- **Mitigate Crisis**
  - $\langle 0, 75 \rangle$

- **Save Hostages**
  - $\langle 10, 30 \rangle$

- **Capture Terrorists**
  - $\langle 20, 100 \rangle$

- **Extract Hostages**
  - $\langle 10, 30 \rangle$

- **Medical Attention**
  - $\langle 10, \infty \rangle$

- **Arrest**
  - $\langle 0, 50 \rangle$

- **Surveil**
  - $\langle 0, 45 \rangle$

Average Domain Size in DCOP: $\infty$
Syntax: \(\langle\text{Earliest Start Time, Latest Start Time}\rangle\)

Example

Mitigate Crisis
\(\langle 0, 75 \rangle\)

- Save Hostages
  \(\langle 10, 30 \rangle\)
  - Extract Hostages
    \(\langle 10, 30 \rangle\)
  - Medical Attention
    \(\langle 10, 30 \rangle\)

- Capture Terrorists
  \(\langle 20, 100 \rangle\)
  - Arrest
    \(\langle 0, 50 \rangle\)
  - Surveil
    \(\langle 0, 45 \rangle\)

Average Domain Size in DCOP: 33.75
Syntax: \( \langle \text{Earliest Start Time, Latest Start Time} \rangle \)

Example

- **Mitigate Crisis**
  \( \langle 0, 75 \rangle \)
  - **Save Hostages**
    \( \langle 10, 30 \rangle \)
    - **Extract Hostages**
      \( \langle 10, 30 \rangle \)
    - **Medical Attention**
      \( \langle 10, 30 \rangle \)
  - **Capture Terrorists**
    \( \langle 20, 100 \rangle \)
    - **Arrest**
      \( \langle 0, 50 \rangle \)
    - **Surveil**
      \( \langle 0, 45 \rangle \)

Average Domain Size in DCOP: 33.75
Syntax: \(\langle\text{Earliest Start Time, Latest Start Time}\rangle\)

**Example**

- **Mitigate Crisis**: \(\langle 0, 75 \rangle\)
- **Save Hostages**: \(\langle 10, 30 \rangle\)
- **Capture Terrorists**: \(\langle 20, 75 \rangle\)
  - **Extract Hostages**: \(\langle 10, 30 \rangle\)
  - **Medical Attention**: \(\langle 10, 30 \rangle\)
  - **Arrest**: \(\langle 0, 50 \rangle\)
  - **Surveil**: \(\langle 0, 45 \rangle\)

Average Domain Size in DCOP: 33.75
Syntax: \( \langle \text{Earliest Start Time, Latest Start Time} \rangle \)

**Example**

- **Mitigate Crisis** \( \langle 0, 75 \rangle \)
  - **Save Hostages** \( \langle 10, 30 \rangle \)
  - **Capture Terrorists** \( \langle 20, 75 \rangle \)
    - **Extract Hostages** \( \langle 10, 30 \rangle \)
    - **Medical Attention** \( \langle 10, 30 \rangle \)
    - **Arrest** \( \langle 0, 50 \rangle \)
    - **Surveil** \( \langle 0, 45 \rangle \)

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Example:

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    - **Arrest**: \( \langle 20, 50 \rangle \)
    - **Surveil**: \( \langle 0, 45 \rangle \)

Average Domain Size in DCOP: 28.75
Syntax: \( \langle \text{Earliest Start Time, Latest Start Time} \rangle \)

**Example**

- **Mitigate Crisis**  \( \langle 0, 75 \rangle \)
  - **Save Hostages**  \( \langle 10, 30 \rangle \)
  - **Capture Terrorists**  \( \langle 20, 75 \rangle \)
    - **Extract Hostages**  \( \langle 10, 30 \rangle \)
    - **Medical Attention**  \( \langle 10, 30 \rangle \)
    - **Arrest**  \( \langle 20, 50 \rangle \)
    - **Surveil**  \( \langle 0, 45 \rangle \)

**Average Domain Size in DCOP:** 28.75
Syntax: $\langle \text{Earliest Start Time, Latest Start Time} \rangle$

**Example**

- **Mitigate Crisis**
  - $\langle 0, 75 \rangle$
  - **Save Hostages**
    - $\langle 10, 30 \rangle$
  - **Capture Terrorists**
    - $\langle 20, 75 \rangle$
      - **Extract Hostages**
        - $\langle 10, 30 \rangle$
      - **Medical Attention**
        - $\langle 10, 30 \rangle$
      - **Arrest**
        - $\langle 20, 50 \rangle$
      - **Surveil**
        - $\langle 20, 45 \rangle$

Average Domain Size in DCOP: 23.75
Syntax: $\langle$Earliest Start Time, Latest Start Time$\rangle$

Example

- Mitigate Crisis: $\langle 0, 75 \rangle$
- Save Hostages: $\langle 10, 30 \rangle$
- Capture Terrorists: $\langle 20, 75 \rangle$
  - Extract Hostages: $\langle 10, 30 \rangle$
  - Medical Attention: $\langle 10, 30 \rangle$
  - Arrest: $\langle 20, 50 \rangle$
  - Surveil: $\langle 20, 45 \rangle$

Average Domain Size in DCOP: 23.75
Constraint Propagation

Idea

Ensure arc consistency by propagating bound information forward down the NLE chains.

Example

\[ M_1 \]
\[ \rightarrow \]
\[ M_2 \]

\[ \text{time} \]
Idea

Ensure arc consistency by propagating bound information forward down the NLE chains.

Example

\[ M_1 \]

\[ M_2 \]

---------- time ----------
Constraint Propagation

Idea
Ensure arc consistency by propagating bound information forward down the NLE chains.

Example

\[ \xymatrix{ & M_1 & \\ M_2 & & \text{Expected duration of } M_1 } \]

We provide a distributed algorithm to perform constraint propagation.
Idea

Ensure arc consistency by propagating bound information forward down the NLE chains.

Example

\[ M_1 \]
\[ M_2 \]

Expected duration of \( M_1 \)

We provide a distributed algorithm to perform constraint propagation.

Sultanik, Modi, and Regli

On Modeling Multi-Agent Task Scheduling
Approach

**Task Scheduling Problem**

**Representation**

**C_TÆMS**

**Algorithms**

**DCOP**

**Schedule**

Optimizations

Measurement

Macro Agent Transport Event-Based Simulator (MATES)

---

Sultanik, Modi, and Regli

On Modeling Multi-Agent Task Scheduling
Motivation
Approach
Summary of Contributions

Measurement and Empirical Analysis
Conclusions and Future Work

Approach

**Task Scheduling Problem**

**Representation**
- C_TÆMS

**Algorithms**
- DCOP

**Schedule**

Optimizations
Measurement

Macro Agent Transport Event-Based Simulator (MATES)

Sultanik, Modi, and Regli

On Modeling Multi-Agent Task Scheduling
Research community provided random scenario generator.\(^a\)

Randomly-generated set of 100 DSC\(_{TAEMS}\) instances

- 4 agents,
- 3–4 windows\(^b\)
- 1–3 agents per window, and
- 1–3 NLE chains.
- Avg. 80 Variables
- Domains of avg. cardinality 124

ADOPT (Modi, Shen, Tambe, & Yokoo, 2005)

MATES (Sultanik, Peysakhov, & Regli, 2005)

Static Network

Cycles & Solvability

\(^a\)http://www.coordin8.net/

\(^b\)“Windows” are tasks whose parent is the task group (i.e., they are tasks at the second layer from the root in the HTN).
Naïve Bounding by itself is insufficient.

95% certainty that Constraint Propagation made an average domain size reduction of 7.62% over those of Bounds Propagation (upper one-sided paired t-test).

Using all three techniques 26% became solvable.
Conclusions

- Presented a mapping from a subset of the C_TÆMS modeling language to an equivalent DCOP.
- Domain bounding heuristics that render solvability.
- Demonstrated efficacy of existing algorithm on mapping.

Future Directions

- Extend DSC_TÆMS to subsume larger subset of C_TÆMS.
- Hierarchical solution.
- Dynamic Re-Scheduling.
- Simulation over disruption-prone Mobile Ad-Hoc Networks (using MATES) → reduce network overhead.
Conclusions

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Thank you for your time and attention!

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Related Work in Multi-Agent Scheduling

Musliner, et al., 2006:  
map C_TAEMS to a MDP;

Smith, et al., 2006:  
address dynamic schedule revision using Simple Temporal Networks;

Phelps and Rye, 2006:  
address this same problem through Generalized Partial Global Planning; and

Cornell C_TAEMS Scheduler:  
offline and omniscient.
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Cornell C_TAEMS Scheduler: offline and omniscient.

Problem: Existing approaches are centralized!
### Measurement and Empirical Analysis

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<th># Windows</th>
<th># NLE Chains</th>
<th>% Soluble Naïve</th>
<th>Avg. # Cycles Naïve</th>
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* This is the default configuration for the C\_TÆMS scenario generator.
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On Modeling Multi-Agent Task Scheduling

Sultanik, Modi, and Regli

Department of Computer Science

Drexel University
Results

Bandwidth Usage (Messages Sent * Hops Per Message) vs. Simulated Time to Optimal Solution

Uncoupled

Coupled

Sultanik, Modi, and Regli

On Modeling Multi-Agent Task Scheduling
Results

![Graph showing Bandwidth Usage (Messages Sent * Hops Per Message) vs. Simulated Time to Optimal Solution with three curves for Uncoupled, Optimized, and Coupled models. The graph includes a detailed inset chart with data points for each category.]

- **Uncoupled**
- **Optimized**
- **Coupled**

**Bandwidth Usage (Messages Sent * Hops Per Message)**

**Simulated Time to Optimal Solution**

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On Modeling Multi-Agent Task Scheduling
Results

5% Bandwidth Reduction with No Additional Computation Time
The problem of finding an optimal schedule is \(\mathcal{NP}\)-hard.
The problem of finding an optimal schedule is \( \mathcal{NP} \)-Hard.

Proof.
Reduction from \( k \)-coloring of graphs. Given a graph \( G = \langle V, E \rangle \), create one agent in \( N \) for each \( v \in V \). Create \( k \) methods associated with each agent, one for each of the \( k \) colors. Therefore, \((\forall a \in N : |\mu^{-1}(a)| = k)\), and \(|M| = k|V|\). Define each method as \( \langle 0, 0, 0 \rangle \), meaning that each method can either be executed at time 0 or it will not be executed at all. The mutex constraints will ensure that each agent can only have at most one method executed...
Proof Sketches

Theorem

*The problem of finding an optimal schedule is \( \mathcal{NP} \)-HARD.*

Proof.

...Create an enables NLE (*i.e.* precedence constraint) between all pairs of methods associated with like colors on adjacent vertices. Since each method has only one feasible start time, these NLEs ensure that two adjacent vertices cannot have two methods of the same color scheduled to execute. Creating this mapping will require \( \binom{n}{2} 2k = O(n^2) \) operations, which is \( \in \mathcal{P} \).
Theorem

The proposed method of naïve domain bounding will not result in suboptimal solutions.
Theorem

The proposed method of naïve domain bounding will not result in suboptimal solutions.

Proof.

The longest possible C$_{TÆMS}$ schedule duration will occur when all of the methods are chosen to execute. A schedule will have a maximal completion time when there exists an enables NLE chain over all of the methods, rooted at the method with maximum earliest start time. There might be an infinite number of optimal solutions to a C$_{TÆMS}$ instance. All optimal schedules, however, will have the same quality as the schedule in which all of the methods are executed, in order, starting with the method with maximum earliest start time. . .
Theorem

*The proposed method of naïve domain bounding will not result in suboptimal solutions.*

Proof.

Let us assume, on the contrary, that the optimal schedule *does not* execute all methods, yet the naïve bounding *does* prune the optimal solution. The only way this can be the case is if some method’s optimal execution time were *greater* than the maximum start time plus the longest possible duration, which would mean such a schedule would be longer than the longest possible duration: a contradiction.